Calculation of the Compression Zone Height in Continuous Thickeners

A method to calculate the compression zone height in continuous thickeners is presented. With this method, it is necessary to know the variations of the pulp-supernatant interface and the sediment height vs. time in a batch test. This method is discussed considering the following aspects: relationship between the settling rate of solids and their concentration in the settling zone; and the compression of solids due to the squeeze transmitted by the upper layers—unbuoyed weight of particles minus force of friction due to the Darcian flow. When the variation of the sediment height vs. time becomes linear, it is possible to calculate the maximum solids concentration which can be reached by sedimentation. The change of the solids matrix permeability and its influence on the method proposed are also analyzed.

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Introduction

Many contributions to the continuous thickener design can be found in literature. Two aspects are mainly considered: the thickener area per unit of solids flow and the compression zone depth. Fitch (1979) presented an up-to-date revision and the different assumptions and methods. Modifications and reinterpretations of the Kynch theory have been presented in some papers, taking into account the compression zone formed on the bottom of the cylinder in a batch test (Tiller, 1981; Fitch, 1983; Font, 1988). These papers have focused their attention on the following aspects:

- The relation between the sedimentation rate and the solids concentration in the solids concentration interval corresponding to the hindered settling zone.
- The calculation of the unit area of a continuous thickener, taking into account the settling velocity-solids concentration relation previously obtained.

The compression zone depth for a continuous thickener, for specified values of any two of underflow concentration, solids throughput rate, and underflow sludge pumping rate can be calculated by equations proposed in literature (Fitch, 1966; Dixon, 1980, 1981; Landman et al., 1988). This requires data on the compressibility and permeability of the sediment as a function of solids concentration. To obtain the latter data from batch tests normally requires extensive experimental work.

Depth, concentration profiles, etc. obtained from batch testing are not normally directly applicable to continuous operation, since average driving forces in batch operation are different from those in the steady-state continuous operation and, moreover, vary with time. In batch sludge thickening, however, there are circumstances, under which batch results are directly applicable to the steady-state continuous operation. This allows, in particular, sludge depth required to be obtained directly from the depth at an appropriate time in a batch test, thus greatly reducing the amount of experimental testing. The method proposed is based on the material and momentum balances, distinguishing the hindered settling zone (where the initial solids concentration is considered to be) from the compression zone. In addition, the method proposed can be applied to suspensions where the permeability, corresponding to the different layers of the compression zone, changes with the residence time of the solids inside the sediment. If this fact is ignored and a permeability-solids concentration relation (obtained at high residence time of solids in the sediment) is used, the calculated value of the compression zone depth can be incorrect.

The following other methods are based on the results of only one batch test for calculation of the compression zone depth in continuous thickeners.

- One method (Coulson et al., 1978) proposes that the time required to concentrate the sediment after it has reached the critical concentration (boundary concentration between settling zone and compression zone) can be determined approximately by allowing a sample of the suspension at its critical composition to settle in a vertical glass tube and by measuring the time taken for the interface between the suspension (or sediment) and the clear liquid to fall to such a level that the concentration is that required in the underflow from the thickener. The use of data so obtained assumes that the average concentration in the sediment in the batch test is the same as that which would be obtained in the thickener after the same time period.

- Another similar method proposes the calculation of the compression zone height from the mean residence time of solids, calculated between the critical point of a batch test and a later point (Foust et al., 1960).
- Merta and Ziolo (1985) have deduced an expression to calculate the compression zone volume, taking into account the assumptions of the previous method.

These methods are not accurate, because in the continuous thickener the various parts of the sediment have been under compression for different times and, therefore, the driving forces are distinct in the batch operation from those in continuous operations. In the batch operation, there is a gradient of solids concentration and, consequently, a mean value cannot be assumed.

Variation of Solids Concentration in the Compression Zone of a Batch Test

Consider a suspension, in which the settling flux vs. solids concentration is varied, Figure 1. In a previous paper, Font (1988) analyzed the supernatant-suspension interface height and the sediment height interface. In accordance with Figure 1 and considering the initial concentration ϕ_{so} of a batch test, the variations of the interface heights are shown in Figure 2.

For initial concentrations different from that indicated in Figure 1, the variations of the interface heights in a batch test would be distinct. [See Koss (1977) for deducing the variation of the supernatant-suspension interface height.] Nevertheless, the conclusions obtained in this paper can be applied for the continuous thickener design when the initial solids concentration is in any zone considered inside the hindered settling range. In Figure 1, a normal settling flux density vs. concentration curve is drawn. This variation corresponds to unflocculated spheres or flocculated suspensions, whose flocs have a constant diameter and density (Michaels and Bolger, 1962). Nevertheless, the method can be applied to suspensions with any hydrodynamics when the initial concentration is inside the hindered settling zone, taking into account the rising of characteristics and discontinuities in accordance with the analysis presented by Fitch (1983).

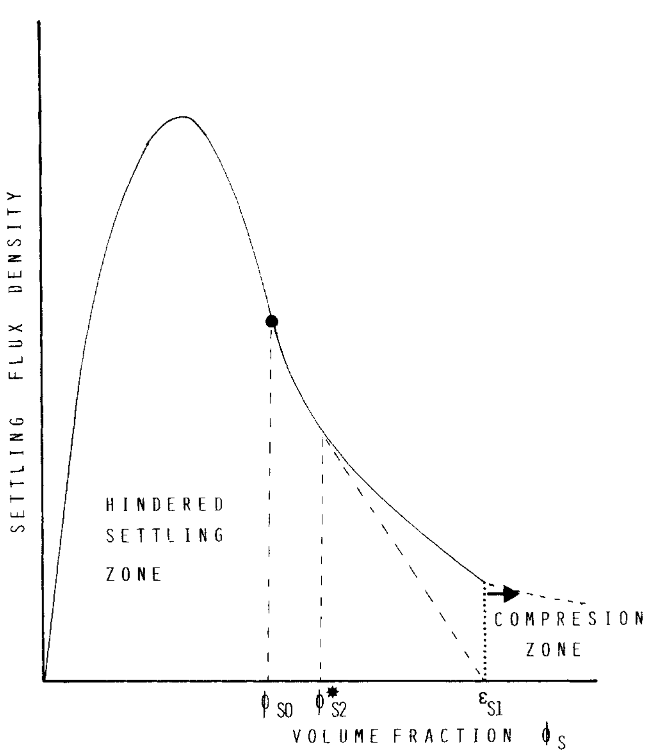

Figure 1. Settling flux density vs. volume fraction of solids.

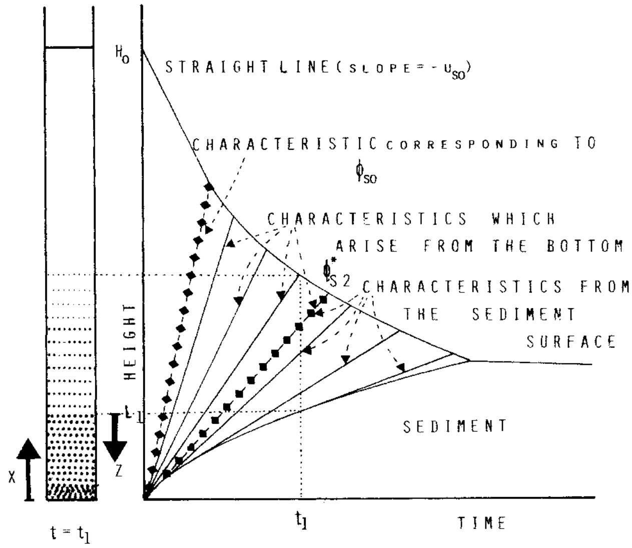

Figure 2. Interface heights vs. time.

According to the relations presented elsewhere (Font, 1988), the following equations were obtained from Kynch's theorems:

$$\frac{dL_1}{dt_1} = \frac{\phi_{s2}(-u_{s2}) - \int_0^{L_1} \frac{\partial \epsilon_s}{\partial t_1} dx}{\epsilon_{s1} - \phi_{s2}} = \frac{\phi_{s2}(-u_{s2}) - \epsilon_{s1}(-u_{s1})}{\epsilon_{s1} - \phi_{s2}} \quad (1)$$

from a balance around the sediment surface, and

$$\frac{dL_1}{dt_1} = -\frac{d[\phi_{s2}(-u_{s2})]}{d\phi_{s2}} \quad (2)$$

from the rising of the characteristics, tangentially to the sediment surface.

Both equations can be applied at any time to the critical point (intersection between the supernatant-suspension interface and the sediment interface).

An analysis of the physical significance of the values ϵ_{su} (underflow solids concentration in a continuous thickener, the unit area of which is calculated from the batch test data) was also presented (Font, 1988). A wider consideration of this analysis leads to interesting conclusions for the calculation of the compression zone height of continuous thickeners. This is presented in this paper.

In the compression zone, the sedimentation rates of the solids depend on their concentrations as well as on the squeeze transmitted by the upper layers (on considering the unbuoyed weight of the upper layers of solids and the force of friction due to the flow of liquid across the solids).

From a momentum balance applied to a differential volume, neglecting inertial effects, the following equation can be deduced (Fitch, 1979; Tiller, 1981):

$$\underbrace{-\frac{\partial p_s}{\partial x} = \frac{dp_s}{d\epsilon_s}\left(-\frac{\partial \epsilon_s}{\partial x}\right)}_{\text{variation of effective pressure}}$$

$$= \underbrace{(\rho_s - \rho)g\epsilon_s}_{\substack{\text{downward force}\\ \text{of the unbuoyed}\\ \text{weight of particles}}} - \underbrace{\frac{\mu(1-\epsilon_s)}{k}[u + (-u_s)]}_{\substack{\text{upward force of friction due}\\ \text{to Darcian flow of the liquid}}} \quad (3)$$

where the effective pressure p_s and the permeability k depend on the volume fraction of solids. This means that the effective pressure is considered to be equal to the compressive yield value, if the hydrodynamic contribution resulting from reluctance of fluid to flow out from between approaching particles is negligible (Fitch, 1979).

Taking into account that the flow by compression of particles $(-u_s)\epsilon_s$ downwards is balanced by the flow of liquid $u(1-\epsilon_s)$ upwards, we deduce that

$$(-u_s)\epsilon_s = u(1-\epsilon_s) \quad (4)$$

From Eqs. 3 and 4

$$\frac{dp_s}{d\epsilon_s}\left(-\frac{\partial \epsilon_s}{\partial x}\right) = (\rho_s - \rho)g\epsilon_s - (\mu/k)(-u_s) \quad (5)$$

where (μ/k) is the specific hydraulic resistance to flow of fluid.

The continuity equation is

$$\frac{\partial \epsilon_s}{\partial t} - \frac{\partial[\epsilon_s(-u_s)]}{\partial x} = 0 \quad (6)$$

In Eq. 6, ϕ_s is the volume fraction of solids corresponding to a layer, the distance of which from the bottom of the cylinder is x.

For a layer of distance z from the sediment interface, which arises with a velocity dL_1/dt_1, we can write

$$\frac{\partial \epsilon_s(z)}{\partial t} = \frac{\partial \epsilon_s}{\partial t} + \frac{\partial \epsilon_s}{\partial x}\frac{dL_1}{dt_1} \quad (7)$$

From Eqs. 6 and 7, taking into account that $x = L_1 - z$, we deduce

$$\frac{\partial \epsilon_s(z)}{\partial t_1} + \frac{dL_1}{dt_1}\frac{\partial \epsilon_s(z)}{\partial z} + \frac{\partial[\epsilon_s(z)(-u_s)]}{\partial z} = 0 \quad (8)$$

Equation 5 can be written as

$$\frac{dp_s}{d\epsilon_s}\left[\frac{\partial \epsilon_s(z)}{\partial z}\right] = (\rho_s - \rho)g\epsilon_s(z) - (\mu/k)(-u_s) \quad (9)$$

From Eqs. 8 and 9

$$\frac{\partial \epsilon_s(z)}{\partial t_1} + \frac{dL_1}{dt_1}\left(\frac{\partial \epsilon_s(z)}{\partial z}\right) + (\rho_s - \rho)g\frac{\partial}{\partial z}\left[\frac{k}{\mu}\epsilon_s^2(z)\right] - \frac{\partial}{\partial z}\left[\frac{k}{\mu}\epsilon_s(z)\frac{dp_s}{d\epsilon_s}\frac{\partial \epsilon_s(z)}{\partial z}\right] = 0 \quad (10)$$

Equation 10 relates the variation of the solids concentration $\epsilon_s(z)$ to the time, taking into account the permeability k and $dp_s/d\epsilon_s$ (in the sediment to the critical point).

In the following sections, these equations are compared with those obtained for continuous thickeners under steady-state conditions.

Variation of Solids Concentration in the Compression Zone of a Continuous Thickener

Figure 3 shows the continuous thickener considered and a normal variation of solids concentration at different levels (Comings et al., 1954; Scott, 1970; Turner and Glasser, 1976; Kos, 1977; Dixon, 1980). ϕ_{so}^c is the volume fraction of solids in the inlet stream and ϵ_{su}^c is the solids concentration at the bottom of the thickener. The concentration of the upper layer of the sediment is ϵ_{s1}. (At this concentration level, the particles, aggregates or flocs are in contact.) The change of solids composition from ϕ_{so}^c to ϕ_{s2}^c above the compression zone takes place in a small zone, as can be observed from Figure 3.

The solids volume flux density of the underflow is

$$(-\theta) = \epsilon_{su}^c(-v_u^c) \quad (11)$$

where v_u^c is the downward velocity of pulp resulting from underflow withdrawal. [Velocities and fluxes are considered negative if the movement of particles is contrary to the X-axis indicated in Figure 3. Consequently, values of $(-v_u^c)$ are positive. The same criterion is applied to the other parameters.]

The area A_u per unit of volumetric flow of solids, which equals $1/(-\theta)$ and the downward velocity v_u^c of pulp resulting from underflow withdrawal can be obtained from a Yoshioka construction as indicated in Figure 4 (Yoshioka et al., 1957; Fitch, 1983; Font, 1988). Note that the inlet solids concentration is in the hindered settling range. For suspensions, whose inlet solids concentrations are inside the compression zone, Yoshioka's construction on considering the maximum values of settling velocity (terminal settling velocity) leads to calculating the maximum throughput rate. It is possible in some cases that the tangency (between the operating line and the settling flux density vs. concentration curve in Figure 4) took place in the compression range, although the inlet solids concentration was in the hindered settling zone. In these cases, Yoshioka's construction also results in the calculation of the maximum throughput rate. The analysis of these cases is more complicated than the way it is presented in this paper, which could lead to another research. Probably, in these more complicated cases, the tangency point could correspond to high solids concentrations, and an increase of solids concentration could be achieved more conveniently by other unit operations—filtration or centrifugation. In this paper, it is considered that the tangency point takes place in the hindered settling zone.

The volume fraction ϕ_{s2}^c, corresponding to the layer just above the compression zone, is that corresponding to a slope

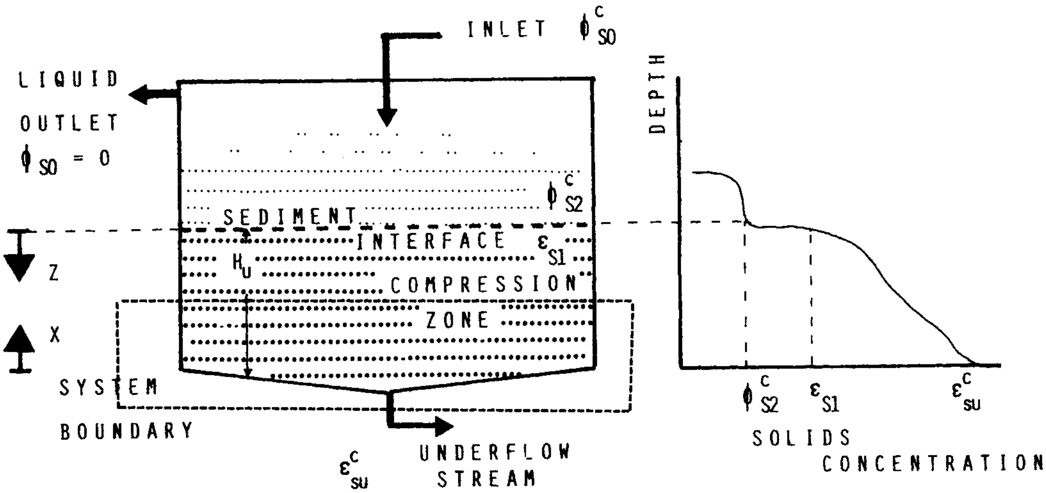

Figure 3. Continuous thickener.

$-d[\phi_s(-u_s)]/d\phi_s|\phi_s = \phi^c_{s2}$, which equals $-\phi^c_{s2}(-u^c_{s2})/(\epsilon^c_{su} - \phi^c_{s2})$ and $-v^c_u$ (Fitch, 1983; Font, 1988).

A method for calculating the unit area A_u has previously been presented (Font, 1988). For different values of ϕ_{s2}, $(-u_{s2})$ and dL_1/dt_1 obtained from a batch test data, values of underflow solids concentration ϵ^c_{su} and of unit area A_u for continuous thickeners can be calculated using the corresponding relations (Font, 1988).

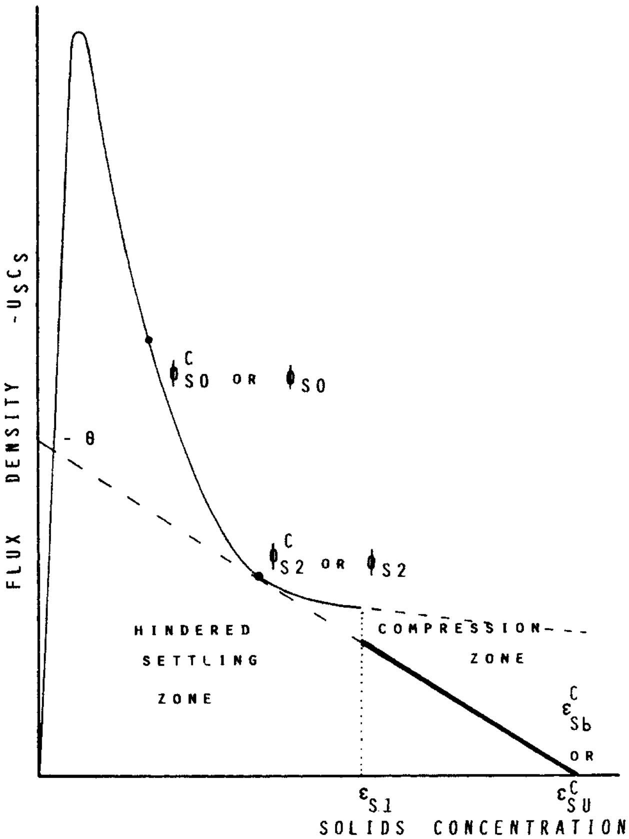

Figure 4. Operative lines in batch test and continuous thickener.

On the other hand, from the balance in a continuous thickener around the system boundary indicated in Figure 3, we deduce

$$\epsilon^c_s[(-v^c_u) + (-u^c_s)] = \epsilon^c_{su}(-v^c_u) = -\theta \tag{12}$$

where $(-u^c_s)$ is the sedimentation velocity with respect to the mixture, whose downward velocity is v^c_u. Consequently, $v^c_u + u^c_s$ is the downward velocity of particles in a layer, the solids concentration of which is ϵ^c_s.

Rearranging Eq. 12

$$\frac{\epsilon^c_s(-u^c_s)}{\epsilon^c_{su} - \epsilon^c_s} = -\frac{\theta}{\epsilon_{su}} = (-v^c_u) \tag{13}$$

Therefore, in Figure 4, the solid straight line is the operating line in the compression zone of a continuous thickener.

The continuity equation can be written as

$$\frac{\partial \epsilon^c_s}{\partial t} - \frac{\partial [\epsilon^c_s((-u^c_s) + (-v^c_u)]}{\partial x} = 0 \tag{14}$$

Under steady state, taking into consideration that v^c_u is constant, from Eq. 14 or differentating Eq. 13 with respect to x, we obtain

$$\frac{d[\epsilon^c_s(-u^c_s)]}{dx} + (-v^c_u)\frac{d\epsilon^c_s}{dx} = 0 \tag{15}$$

From a momentum balance applied to a differential volume, neglecting inertial effects, the following equation can be deduced:

$$\frac{dp_s}{d\epsilon_s}\left(-\frac{\partial \epsilon^c_s}{\partial x}\right) = (\rho_s - \rho)g\epsilon^c_s - \frac{\mu(1-\epsilon^c_s)}{k}[u^c + (-u^c_s) + (-v^c_u)] \tag{16}$$

Taking into account that the mean velocity of the solids + liquid mixture across the continuous thickener is v_u, we deduce that

$$v_u^c = u^c(1 - \epsilon_s^c) + (u_s^c + v_u^c)\epsilon_s^c \tag{17}$$

From Eqs. 16 and 17, and taking into account that the distance z of a layer from the sediment interface equals $H_u - x$, we can write

$$\frac{dp_s}{d\epsilon_s}\left[\frac{\partial \epsilon_s^c(z)}{\partial z}\right] = (\rho_s - \rho)g\epsilon_s^c(z) - \frac{\mu}{k}(-u_s^c) \tag{18}$$

This equation is equivalent to Eq. 3.

Combining Eqs. 15 and 18, and taking into account that $dz = -dx$

$$(-v_u^c)\frac{d\epsilon_s^c(z)}{dz} + (\rho_s - \rho)g\frac{d}{dz}\left[\frac{k}{\mu}\epsilon_s^{c2}(z)\right] - \frac{d}{dz}\left[\frac{k}{\mu}\epsilon_s^c(z)\frac{dp_s}{d\epsilon_s}\frac{\partial \epsilon_s^c(z)}{\partial z}\right] = 0 \tag{19}$$

From the solution of this equation, and considering that when z equals zero and the solids concentration equals ϵ_{s1}, the value of H_u (zone compression height in the thickener corresponding to an underflow solids concentration ϵ_{su}) can be calculated.

Analysis of the Parameters $dp_s/d\epsilon_s$ and k

Equation 9 corresponding to batch tests or Eq. 18 corresponding to continuous thickeners can be written as

$$(\rho_s - \rho)g\epsilon_s = \frac{dp_s}{d\epsilon_s}\frac{\partial \epsilon_s(z)}{\partial z} + \frac{\mu}{k}(-u_s) \tag{20}$$

where all the terms are positive.

In the settling zone, the particles do not contact one another and $dp_s/d\phi_s = 0$. The permeability k can be calculated by the following equation obtained from Eq. 20 dropping the corresponding term,

$$k = \frac{\mu(-u_s)}{(\rho_s - \rho)g\phi_s} \tag{21}$$

In the compression zone of some suspensions, it is possible that the values of k are so high that in every solids concentration the term $(dp_s/d\epsilon_s)(\partial\epsilon_s(z)/\partial z)$ is much greater than the term $(\mu/k)(-u_s)$. Under this assumption, Eq. 20 becomes

$$(\rho_s - \rho)g\epsilon_s = \frac{dp_s}{d\epsilon_s}\frac{\partial \epsilon_s(z)}{\partial z} \tag{22}$$

In this case the compression of the solids is controlled only by the variation of the effective pressure. The influence of the force of friction due to the Darcian flow is very small or nil. This occurs when values k of permeability are very high.

Design of Continuous Thickeners from Batch Test Data

In the design of continuous thickeners, three different situations, Case I, II and III, are considered.

Case I: very high permeability $[(dp_s/d\epsilon_s)(\partial\epsilon_s(z)/\partial z) \gg (\mu/k)(-u_s)]$

Consider the data obtained from a batch test. When the values of permeability corresponding to the solids concentration in the range of the compression zone are very high, Eq. 22 obtained from a momentum balance in a batch test can be expressed as:

$$\frac{\partial \epsilon_s(z)}{\partial z} = (\rho_s - \rho)g\epsilon_s(z)\frac{d\epsilon_s}{dp_s} \tag{23}$$

Value ϵ_{s1} is the volume fraction of solids when the particles, aggregates or flocs, are in contact. Consequently, in a batch test the compression zone extends from the sediment surface (volume fraction of solids ϵ_{s1} just under the sediment surface) to the bottom of the cylinder. Taking into account that $dp_s/d\epsilon_s$ depends only on the solids concentration $\epsilon_s(z)$, we deduce that $\partial\epsilon_s(z)/\partial z$ also depends only on the solids concentration $\epsilon_s(z)$. For z equals zero (top of the sediment) at any time t_1, the volume fraction of solids at the top of the sediment equals ϵ_{s1}. Consequently, taking into consideration that the limit $z = 0$, $\epsilon_s(z) = \epsilon_{s1}$ is always the same for any time t_1; from the integration of Eq. 23 we deduce that $\epsilon_s(z)$ is only a function of the distance z to the top of sediment. If that occurs, Eq. 8 becomes

$$\frac{dL_1}{dt_1}\frac{d\epsilon_s(z)}{dz} + \frac{d[\epsilon_d(z)(-u_s)]}{dz} = 0 \tag{24}$$

Dropping dz in Eq. 24

$$\frac{d[\epsilon_s(z)(-u_s)]}{d\epsilon_s(z)} = -\frac{dL_1}{dt_1} \tag{25}$$

Equation 25 implies that the points $[\epsilon_s(z), \epsilon_s(z)(-u_s)]$ in a diagram, as shown in Figure 4, are in a straight line from the point $(\epsilon_{sb}, 0)$ to $[\epsilon_{s1}, \epsilon_{s1}(-u_{s1})]$ (ϵ_{sb} is the volume fraction of solids at the bottom).

The unit area A_u of a continuous thickener can be calculated from the data of a batch test when $-v_u^c$ equals dL_1/dt_1 and the solids concentration just above the sediment in the batch test and in the continuous thickener are identical (Font, 1988). In the case considered in this section, for a continuous thickener and taking into account that $dx = -dz$, Eq. 22 can be rewritten as

$$\frac{d\epsilon_s^c}{dz} = (\rho_s - \rho)g\epsilon_s^c\frac{d\epsilon_s}{dp_s} \tag{26}$$

Two interesting conclusions can be obtained when $-v_u^c$ (velocity of pulp resulting from underflow withdrawal under steady state) equals dL_1/dt_1 (rising velocity of the sediment interface in a batch test at time t_1):

1. In Figure 4 we observe that the variations are identical. Consequently ϵ_{sb} (volume fraction of solids at the bottom of the cylinder in a batch test) equals ϵ_{su}^c (underflow volume fraction of solids in the continuous thickener).

2. As Eqs. 23 and 26 are identical, the compression zone height H_u of the thickener is equal to the sediment height L_1 in a batch test.

Consequently, in this case, the unit area A_u and the compression zone height H_u of a continuous thickener can be calculated from a batch test using Eqs. 18 and 19 of a previous paper (Font, 1988) and considering that H_u equals L_1 at the time t_1 taken in the batch test.

Note that, in this case, the point of tangency is the hindered settling zone. The settling line can be extended into the compression zone by way of Eq. 21, using the values of terminal settling velocity (velocity at which the net gravitational force on the particles is balanced by drag force and is related to permeability by Eqs. 21).

For a suspension, under the assumption considered in this case, the sediment height after the critical point remains constant as is shown in Figure 5. If there is a considerable change in the sediment height after the critical point. Cases II and III must be considered.

In Figure 5, lines of equal solids concentration are also plotted. They are equidistant to the curve of the sediment height. The value ϵ_s of each curve can be calculated by Eq. 18 of the previous paper (Font, 1988) and by using the values ϕ_{s2}, $-u_{s2}$ and dL_1/dt_1 for $t = t_1$. The curve that arises from the bottom remains equidistant to the sediment interface so that we can draw the lines of equal solids concentration.

Case II: similarity in the values of $(dp_s/d\epsilon_s)\,[\partial\epsilon_s(z)/\partial z]$ *and* $(u/k)(-u_s)$

Equations 10 and 19 relate the solids concentration $\epsilon_s(z)$ with the distance z for the compression zone of a batch test and of a continuous thickener, respectively. The unit area of A_u of a continuous thickener can be calculated by Eqs. 18 and 19 of the previous paper (Font, 1988) from the batch test data when $-v_u^c$ equals dL_1/dt_1. Comparing Eq. 10 (for a batch test) with Eq. 19 (for continuous thickener), we can observe two differences:

1. In Eq. 19 there is not an equivalent term to $\partial\epsilon_s(z)/\partial t_1$.
2. In Eq. 10 the term dL_1/dt_1 varies with the change of time t_1, as can be seen in Figure 2.

Meanwhile, in Eq. 19 $-v_u^c$ is constant (although this parameter equals dL_1/dt_1 at time t_1 considered for the calculation of the unit area). In the batch test, therefore, the solids concentration also depends on time t_1 and the term $\partial\epsilon_s(z)/\partial t_1$ cannot be dropped.

In Figure 6, divergent curves of constant concentration in the compression zone of a batch test can be observed. This divergence is due to the fact that the solids at a given concentration will settle faster as the sediment builds up; therefore, there is more hydrodynamic support of the solids (due to the upward force of friction between solids and fluid) and a lower increase rate of concentration with depth than when the concentration first appears at the bottom of cylinder in a batch test. Consequently, the distance between two lines of equal concentration increases during the batch test in the sediment from $t = 0$ to the time corresponding to the critical point (while solids are being fed into the sediment).

Sometimes, it is possible to observe that the curve $L_1 = f(t_1)$ nearly becomes a straight line after some time as is shown in Figure 6. After the point (t_1^a, L_1^a) the variation of the sediment height is linear with respect to time t_1 (to the critical point).

Taking into consideration that the distance between lines of equal concentration increases when the sedimentation velocities $(-u_s)$ are greater, two conclusions can be deduced when dL_1/dt_1 becomes constant:

1. Lines of equal concentration become parallel to the variation $L_1 = f(t_1)$.

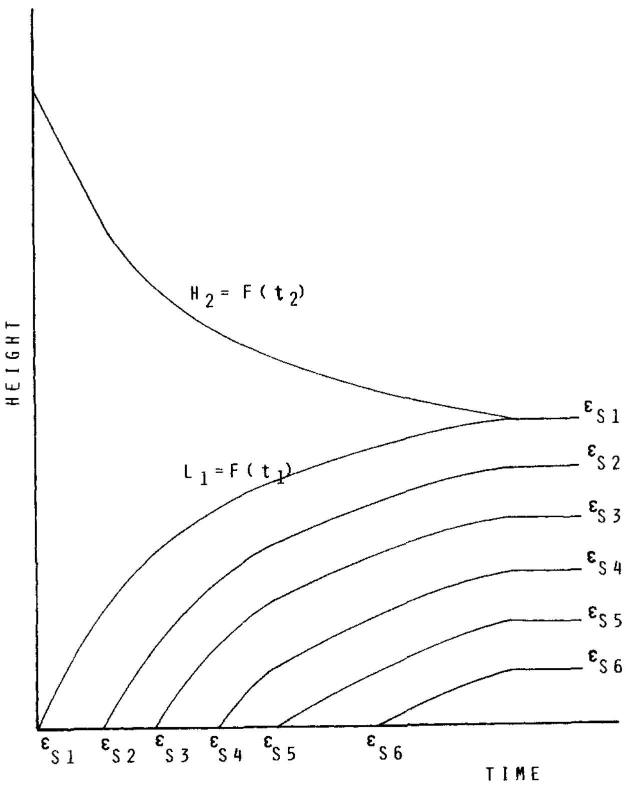

Figure 5. Interface heights vs. time in suspensions Case I.

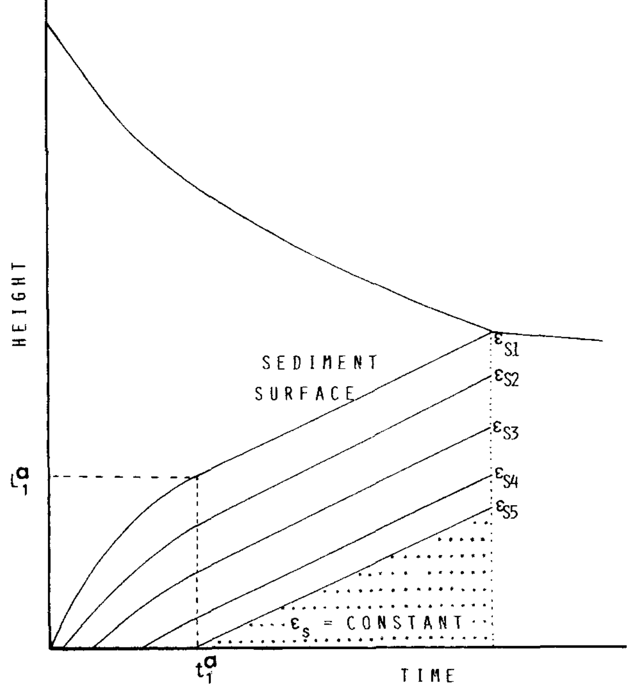

Figure 6. Interface heights vs. time in suspensions Case II.

2. A zone with constant concentration arises from the bottom of the cylinder with a velocity equal to dL_1/dt_1.

Consequently for times greater than t_1^a, the solids concentration $\epsilon_s(z)$ depends only on the distance z to the top of the sediment, and Eq. 10 becomes

$$\frac{dL_1}{dt_1}\frac{d\epsilon_s(z)}{dz} + (\rho_s - \rho)g\frac{d}{dz}\left[\frac{k}{\mu}\epsilon_s^2(z)\right] - \frac{d}{dz}\left[\frac{k}{\mu}\epsilon_s(z)\frac{dp_s}{d\epsilon_s}\frac{d\epsilon_s(z)}{sz}\right] = 0 \quad (27)$$

where dL_1/dt_1 is constant.

Comparing Eq. 27 (batch test when dL_1/dt_1 is constant) with Eq. 19 (continuous thickener), we deduce that they are identical if $-v_u^c$ (downward velocity resulting from underflow withdrawal) equals dL_1/dt_1 [slope of the variation $L_1 = f(t_1)$ when this variation becomes linear]. Consequently, for the design of a continuous thickener, where the underflow solids concentration ϵ_{su} is calculated for the value dL_1/dt_1 (constant obtained at great values of time t_1), the unit area A_u can be calculated by Eqs. 18 and 19 of the previous paper (Font, 1988). The minimum compression zone height H_u at the continuous thickener (considering that Eqs. 27 and 19 are identical) equals L_1^a (sediment height when dL_1/dt_1 becomes constant). The value ϵ_{su} considered here is equal to the solids concentration ϵ_{sb}^{max} at the bottom of the cylinder when dL_1/dt_1 becomes constant. This value ϵ_{sb}^{max} is the maximum which can be obtained by compression of solids. If the aim of the continuous thickener is to obtain the maximum solids concentration, it is possible to estimate this maximum concentration and the dimensions of the continuous thickener by the procedure presented above.

In this discussion, a maximum concentration of sludge is assumed. While this cannot be strictly correct, most flocculated sediments decreases in compressibility as concentration increases and become practically incompressible as the close-packed concentration is approached. Hence, assuming that there is, in fact, a definite maximum concentration, it is clear that a constant graded compression zone will be formed above a uniform zone of increasing depth. Furthermore, once these zones are formed in batch settling, the constant gradient zone will be directly comparable to the graded zone in steady-state continuous thickening when the underflow concentration is ϵ_{sb}^{max} as can be deduced from the previous mathematical discussion.

When the curve $L_1 = f(t_1)$ does not become linear or an underflow with solids concentration less than the maximum which can be reached (when dL_1/dt_1 becomes linear) is required, the procedure presented as follows can be used to estimate the compression zone height.

In Figure 7, lines of equal concentration corresponding to two situations are presented: a) the solid lines correspond to a real situation of the layers with constant solids concentration; and b) the dashed lines correspond to a hypothetical situation where it is assumed that the solids concentration (in the sediment) depends only on the distance z to the top of the sediment (as it occurs in Case I). The hypothetical situation corresponds to a fictional suspension with the same hydrodynamic behavior as the real suspension in the hindered settling zone (same settling velocity-solids concentration relation) but with another behavior pattern in the compression range of concentrations (high values of permeability as Case I) which can lead to the same variation $L_1 = f(t_1)$ as that really obtained.

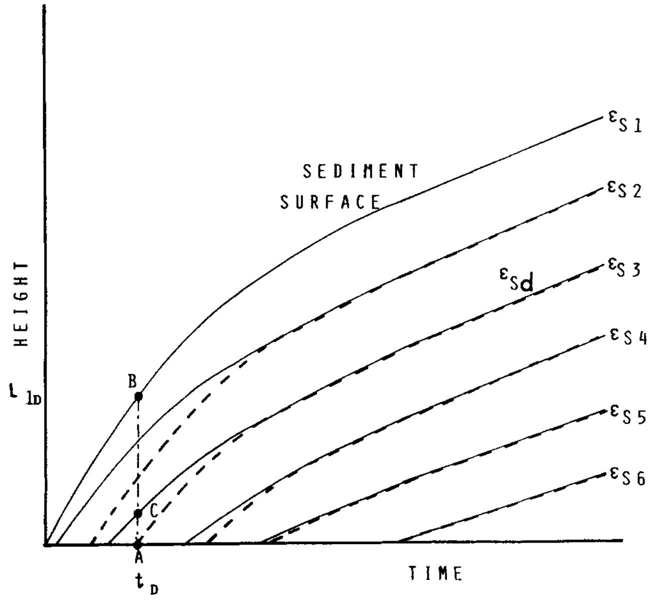

Figure 7. Lines of constant solids concentration in sediment.

Consider the situation indicated at time t_d in Figure 7. The sediment height is AB. The value ϵ_{sd}, solids concentration calculated by Eq. 18 of a previous paper (Font, 1988) or from Eq. 36 of this paper, can be the concentration at the bottom of the cylinder at time t_{1d} if the solids concentration depended only on the distance z to the top of the sediment. Really at time t_d, the layer, whose solids concentration is ϵ_{sd}, is at point C above the bottom. The solids concentration of the layer just above the bottom is ϵ_{se}, which is greater than ϵ_{sd}.

Plots of the flux density $\epsilon_s(-u_s)$ vs. the solids concentration ϵ_s, for the real situation (solid line) and for the hypothetical situation (dashed line) at a time such as t_d, are shown in Figure 8.

For the hypothetical situation (ϵ_s depends only on the distance z), the variation of $\epsilon_s(-u_s)$ vs. ϵ_s is linear as is shown in Figure 4. For the actual situation the following analysis can be done:

The arising velocity of a layer of constant concentration is:

$$v = \frac{dx}{dt} = -\frac{\partial\epsilon_s/\partial t}{\partial\epsilon_s/\partial x} \quad (28)$$

From Eq. 28 and the continuity equation (Eq. 6), we deduce that

$$v = \frac{\partial[\epsilon_s(-u_s)]/\partial x}{\partial\epsilon_s/\partial x} \quad (29)$$

At time t_d we can write

$$v_{t_d} = \frac{\left.\frac{\partial[\epsilon_s(-u_s)]}{\partial x}\right|_{t=t_d}}{\left.\frac{\partial\epsilon_s}{\partial x}\right|_{t=t_d}} = \frac{\left.\frac{\partial[\epsilon_s(-u_s)]}{\partial\epsilon_s}\right|_{t=t_d}\left.\frac{\partial\epsilon_s}{\partial x}\right|_{t=t_d}}{\left.\frac{\partial\epsilon_s}{\partial x}\right|_{t=t_d}} = \left.\frac{\partial[\epsilon_s(-u_s)]}{\partial\epsilon_s}\right|_{t=t_d} \quad (30)$$

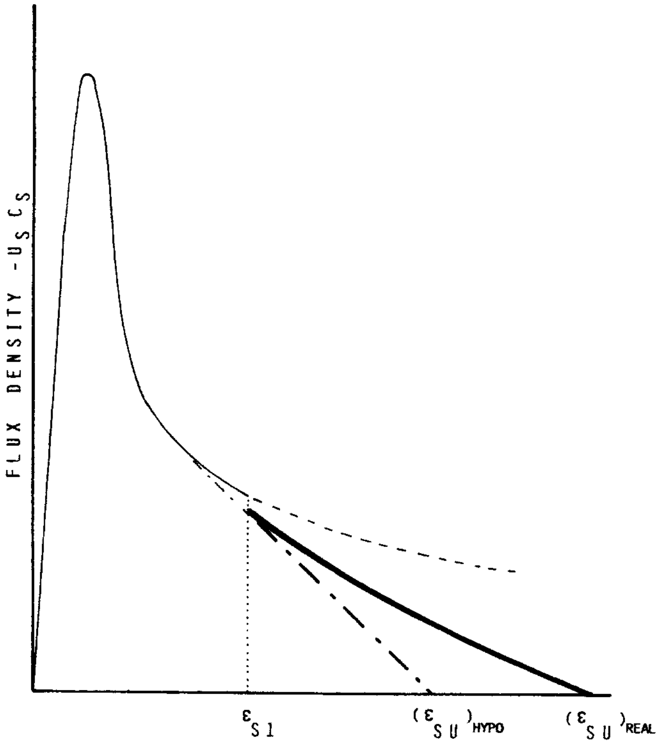

Figure 8. Flux density vs. solids concentration in suspensions Case II.

Equation 30 indicates that at time t_d the slope $\partial\epsilon_s(-u_s)/\partial\epsilon_s$ equals v_{td}, which also equals the slope of the tangent to lines of constant concentration. From Figure 6 we observe that the slopes $v = dx/dt$ at any time are less than dL_1/dt_1 (slope of the sediment height), except for the top of the sediment, in which the solids concentration is ϵ_{s1} and the slope is logically dL_1/dt_1. The slopes $\partial\epsilon_s(-u_s)/\partial\epsilon_s$, therefore, are always less than dL_1/dt_1 (except for ϵ_{s1}). Consequently, the variation of the flux density $\epsilon_s(-u_s)$ vs. solids concentration ϵ_s for the real situation must be that shown in Figure 8. Therefore, for each value of solids concentration ϵ_s, $-u_s|_{real}$ is greater than the corresponding value $-u_s|_{hypo}$ of the hypothetical situation. From Eq. 5 we deduce that

$$\left(-\frac{\partial\epsilon_s}{\partial x}\right)\frac{dp_s/d\epsilon_s}{(\rho_s-\rho)g_s-(\mu/k)(-u_s)}\bigg|_{t=t_d} = 1 \tag{31}$$

The distance CB, in Figure 7, at time t_d can be expressed as

$$CB = \int_{x_C}^{x_B} dx = \int_{\epsilon_{s1}}^{\epsilon_{sd}} \frac{\frac{dp_s}{d\epsilon_s}\,d\epsilon_s}{(\rho_s-\rho)g\epsilon_s-(\mu/k)(-u_s)|_{t=t_d}} \tag{32}$$

For a continuous thickener, in which a volume fraction of solids ϵ_{sd} must be reached in the underflow stream, the compression zone height H_u can be calculated by

$$H_u = \int_{\epsilon_{s1}}^{\epsilon_{sd}} \frac{\frac{dp_s}{d\epsilon_s}\,d\epsilon_s^c}{(\rho_s-\rho)g\epsilon_s^c-(\mu/k)(-u_s^c)} \tag{33}$$

Equation 33 was deduced by Fich (1966) and has been used in some papers (Dixon, 1980; Landman et al., 1988). A sludge plug flow is assumed in the continuous thickener and funneling effects are ignored.

For a continuous thickener with underflow solids concentration ϵ_{sd}, the operating line coincides with the dashed line of Figure 8 (corresponding to the hypothetical situation, Case I). This means that for each value of solids concentration between ϵ_{s1} and ϵ_{sd}, taking into account that values $-u_s$ in the batch test are greater than values $-u_s^c$ for the continuous thickener (equivalent to the hypothetical situation), the denominator in Eq. 33 inside the integration sign is greater than the denominator in Eq. 32. Consequently, the result of the integration H_u in Eq. 33 is less than CB in Eq. 32, but CB is less than AB in Figure 7. Therefore, it can be concluded that the necessary compression zone height in a continuous thickener with underflow solids concentration ϵ_{su}^c is less than the sediment height in a batch test, whose data have been used for calculating the unit area A_u corresponding to ϵ_{su}^c. The design of the continuous thickener, therefore, can be determined with this height (AB in the example considered). This height is greater than that necessary.

Case III: variations in particle arrangement in the compression zone

The discussion of Cases I and II is based on the usual assumption that the solids concentration achieved in the compression zone depends on the effective pressure. However, this is not exactly true: variations in particle packing arrangement can have a significant effect, and there is also a time delay in the compression process which is usually neglected (Dixon, 1980). This effect can be more important at the beginning of a batch test, where channels for fluid upflow are not formed. The previous analysis and the following conclusions continue to be valid if the situation of the lines of equal solids concentration in the sediment is similar to that shown in Figure 7. The time delay in the compression process will approximate the situation of the lines shown in Figure 7: the real ones and the equidistant to the sediment surface. The methods based on the determination of the parameters $dp_s/d\epsilon_s$ and the permeability k as functions of the solids concentration for the integration of Eq. 33 can lead to undercalculation of the compression zone depth required in a continuous thickener, if the delay in the compression process is ignored. Nevertheless, in the batch test, the rising of the sediment takes place according to the material and momentum balances, including logically the variations in arrangement of particles in the sediment. Consequently, the method proposed for the design of continuous thickeners can be adequate when there are great changes in the structure of the solids matrix in the sediment.

Conclusion

From a batch test, where the supernatant-pulp interface and the sediment interface can be observed visually or with the aid of radiation absorption instruments, the area per unit of solids volumetric flow and the compression zone height of continuous thickeners can be calculated as shown below. The method is based on the following assumptions or considerations:

1. In the settling zone, the sedimentation rate depends only on the solids concentration.
2. The inertial effects in the settling zone have a small influence.

3. The discontinuities between zones of different solids concentration take place in a very thin zone.

Using the experimental data of some batch tests, with different initial heights or solids concentration, these assumptions can be tested (Font, 1988). If the properties of the dispersion, such as the surface charge and particle polidispersity, affected the behavior of the suspension in a different way, the calculated settling rate-solids concentration relation would be distinct.

4. In the compression zone, the solids concentration depends on the pressure transmitted by the upper layers (unbuoyed weight minus resistance to fluid motion). A sludge plug flow is assumed in this zone.

5. Funneling effects in the continuous thickener design are not considered. Consequently, an additional height to the compression zone calculated must be taken into consideration. Nevertheless, this assumption does not invalidate the general method proposed; on the contrary, this method can be useful in analyzing the influence of the funneling effects.

The procedure proposed is based on the balances only, taking into account the assumptions cited before. From the data obtained from a batch test performed in a cylinder with constant section and at initial constant concentration ϕ_{so}, a plot of the interface's heights vs. time must be obtained (Figure 9). From the variation of the discontinuities heights in batch testing, it is possible to roughly estimate the situation of the point corresponding to the initial conditions in the settling flux density vs. concentration plot, when considering the analysis carried out by Fitch (1983) with respect to the stability of discontinuities and rising of characteristics. In the discussion performed in this paper, a situation as that shown in Figure 1 has been assumed. Nevertheless, the same conclusions can be drawn for any situation of the initial conditions in a settling flux density vs. concentration curve. The procedure is:

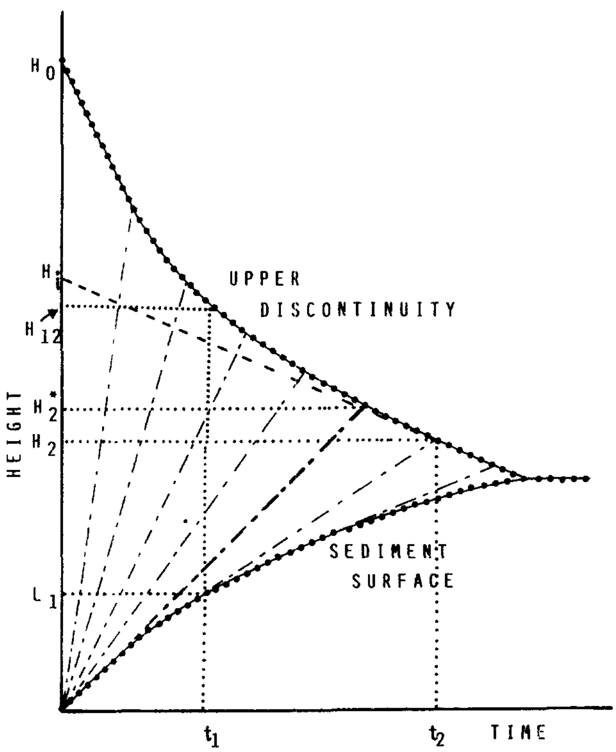

Figure 9. Notation in the method proposed.

Step 1. Calculate the volume fraction of solids ϵ_{s1} corresponding to the limit between compression zone and settling zone by the equation

$$\epsilon_{s1} = \frac{\phi_{so} H_o}{H_2^*} \tag{34}$$

See Figure 9 and Font (1988).

Step 2. If the underflow solids concentration in the continuous thickener is less than ϵ_{s1}, the Talmadge and Fitch method (1955) can be used to calculate the unit area A_u. In this case, the height of the compression zone is zero. Theoretically the change in composition between the inlet solids concentration and the underflow solids concentration (less than ϵ_{s1}) takes place in a very small zone; therefore, the minimal height of the thickener is needed to perform this operation.

Step 3. If the suspension is to be concentrated to values of volume fraction of solids greater than ϵ_{s1} (normal situation), the following steps must be taken:

a. Draw tangent lines to the sediment surface until the upper interface, as is shown in Figure 9.

b. For different values of t_1 and t_2, between $t_1^* = 0$, $t_2 = t_2^*$ and greater values, calculate the volume fraction of solids ϕ_{s2} by the equation

$$\phi_{s2} = \frac{\phi_{so} H_o}{H_{12} - L_1} \exp\left(-\int_0^{t_1} \frac{1}{t_2 - t_1} dt_1\right) \tag{35}$$

When in Figure 9 the upper interface is a straight line from the initial H_o to the critical point, the solids concentration in the region above the sediment surface equals the initial solids concentration. This is due to the fact that there are no characteristics different from those corresponding to the initial solids concentration. The application of Eq. 35 in these cases leads to the same result: values ϕ_{s2} equals ϕ_{so}. Consequently, Eq. 35 can be used for all of the above.

The underflow solids concentration of a thickener can be calculated as (Font, 1988):

$$\epsilon_{su} = \phi_{s2}\left(1 + \frac{-u_{s2}}{dL_1/dt_1}\right) \tag{36}$$

where ϕ_{s2} is obtained by Eq. 35, $-u_{s2}$ $(= -dH_2/dt_2)$ is the slope of the tangent to the curve $H_2 = f(t_2)$ at the point (t_2, H_2). The value of dL_1/dt_1 is the slope of the tangent to the curve $L_1 = f(t_1)$ at the point (t_1, L_1). The unit area A_u, required for obtaining an underflow stream with solids concentration ϵ_{su}, can be calculated by (Font, 1988):

$$A_u = \frac{1}{(dL_1/dt_1)\epsilon_{su}} \tag{37}$$

The required height of the compression zone in the continuous thickener is less than the value of L_1 (sediment height) corresponding to the time t_1 taken as reference. Consequently, the value of L_1 can be proposed as the compression zone height for the design of continuous thickeners.

Considering different values of t_1, the corresponding values of ϕ_{s2}, $-u_{s2}$, dL_1/dt_1 can be calculated, and then the values of ϵ_{su}

and A_u can be obtained. Rearranging the data for different values of the underflow solids concentration ϵ_{su}, the corresponding values of unit area A_u and compression zone height L_1 can be deduced.

c. When the curve of $L_1 = f(t_1)$ becomes linear, the equations presented above can be used to calculate the maximum solids concentration which can be obtained in a sedimentation process and the parameters of design corresponding to a continuous thickener. The value of t_1 considered corresponds to the point, at which the curve of $L_1 = f(t_1)$ becomes linear. In this case, the value of t_2 is that corresponding to the critical point. Taking into account all the tangent lines drawn from the sediment surface, Eq. 35 must be used to calculate the corresponding value of ϕ_{s2}. The value of $-u_{s2}$ is calculated from the tangent to the curve $H_2 = f(t_2)$ just before the critical point. The value of dL_1/dt_1 is the slope of the curve when it becomes linear. By Eq. 36, the value of ϵ_{su} can be calculated. This value represents the maximum solids concentration which can be reached by a sedimentation process. The corresponding unit area A_u can be calculated by Eq. 37. The minimal compression zone height required in a continuous thickener is the value of L_1 (sediment height) at the point just where the curve $L_1 = f(t_1)$ becomes linear.

Inasmuch as the linearity of $L_1 = f(t_1)$ is difficult to observe exactly, the maximum value of solids concentration represents a limit which can be reached easily with compression zone heights similar to those obtained in batch tests. Greater solids concentrations could be obtained with other unit operations: centrifugation or filtration.

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Notation

A_u = unit area of continuous thickener, $m^2/(m^3$ solids/s)
g = gravity acceleration, m/s^2
H_i = intercept height of tangent to curve $H_2 = f(t_2)$ on X axis, m
H_o = initial height of suspension, m
H_u = height of the compression zone in continuous thickeners, m
H_1, H_2 = heights of descending interface at t_1 and t_2, m
H_{12} = intercept height between parallel line to X axis that passes through (t_1, L_1) and tangent to curve $H_2 = f(t_2)$, m
k = permeability, m^2
L_1 = sediment height at time t_1, m
L_1^a = value of L_1 when dL_1/dt_1 becomes constant, m
L_{1d} = value of L_1 at point D, m
p_s = effective pressure, N/m^2
t = time, s
t_1, t_2 = values of t at intersection of characteristic with sediment (t_1) and upper interface (t_2)
t_1^a = value of t_1 when dL_1/dt_1 becomes constant, s
t_d = value of t_1 at point D, s
u = velocity of water upwards, m/s
u_s = sedimentation velocity of solids, m/s
u_{s1} = sedimentation velocity of solids at the top of sediment, m/s
u_{s2} = value of u_s at upper interface corresponding to a characteristic that arises from the sediment surface, m/s
v = rising velocity of the layer of constant solids concentration, m/s
v_u^c = downward velocity of pulp resulting from underflow withdrawal, m/s
x = distance down column, m
z = distance from sediment top, m

Greek letters

ϵ_s = volume fraction of solids in sediment
ϵ_{s1} = value of ϵ_s at sediment surface
ϵ_{su}^c = underflow volume fraction of solids
ϵ_{sb} = value of ϵ_s at the bottom of column
ϵ_{sd} = value of ϵ_s at point D
θ = maximum thickener volumetric flux density of solids, m^3 solids/$(m^2 \cdot s)$
μ = viscosity of fluid, kg/(m · s)
ρ = fluid density, kg/m^3
ρ_s = solids density, kg/m^3
ϕ_s = volume fraction of solids
ϕ_{so} = initial suspension concentration
ϕ_{s2} = value of ϕ_s corresponding to layers that arise from sediment
ϕ_{s2}^c = volume fraction of solids just above the sediment in a continuous thickener

Symbols

$(\ )^*$ = value related to characteristic that arises from the bottom of column tangentially to sediment curve
$(\ )^c$ = value related to continuous thickener

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